Lecture 13

Photonic Signals and Systems

- **An Introduction**
- **By**
- **Nabeel A. Riza ***

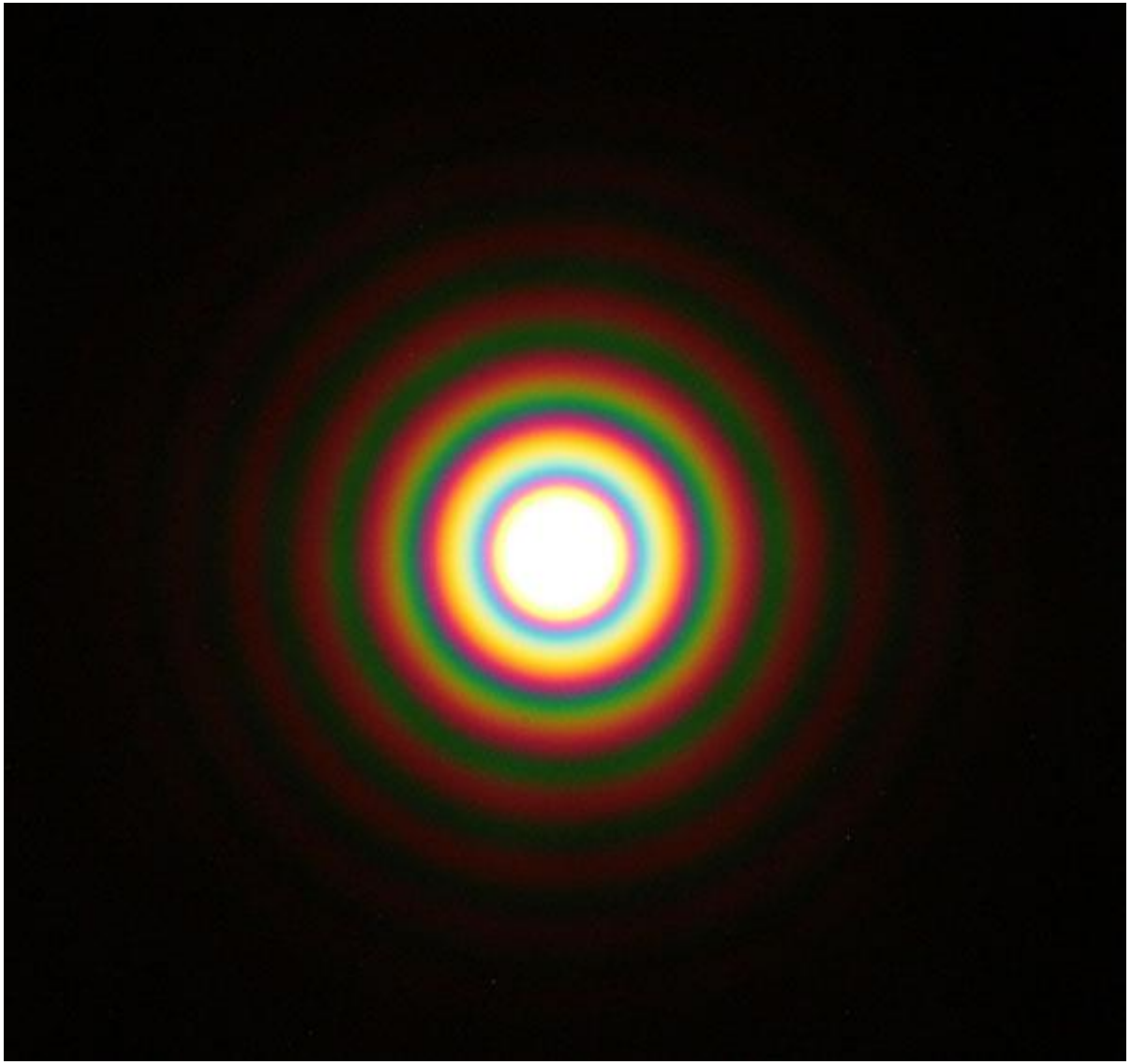
- *Text Book Reference: N. A. Riza, Photonic Signals and Systems – An Introduction, McGraw Hill, New York, 2013.*

Lecture 13
Overview

Topics:

- EM wave Diffraction is the ideal mathematical phenomena of interference (discrete summations of fields), but when considering finite size physical apertures that interact with interacting EM waves
- Physical and Mathematical (Continuous summations via Integral) Hugyens-Fresnel representation of Diffraction using both amplitude and phase of the secondary wavelets at the diffracting aperture
- Fraunhofer (Far field) diffraction – the spatial Fourier transform via Freespace Propagation – e.g., for a 1-D aperture-Rect Function in space.
- Fraunhofer (Far field) diffraction – the spatial Fourier transform via a Spherical Optical Lens
- Fresnel (Near Field) Diffraction – The Linear Shift-Invariant (LSI) optical system via Freespace Propagation with 2-D impulse response given by a quadratic spatial phase function-Spherical Wavelet. The convolution operation via Near-field Freespace optical propagation

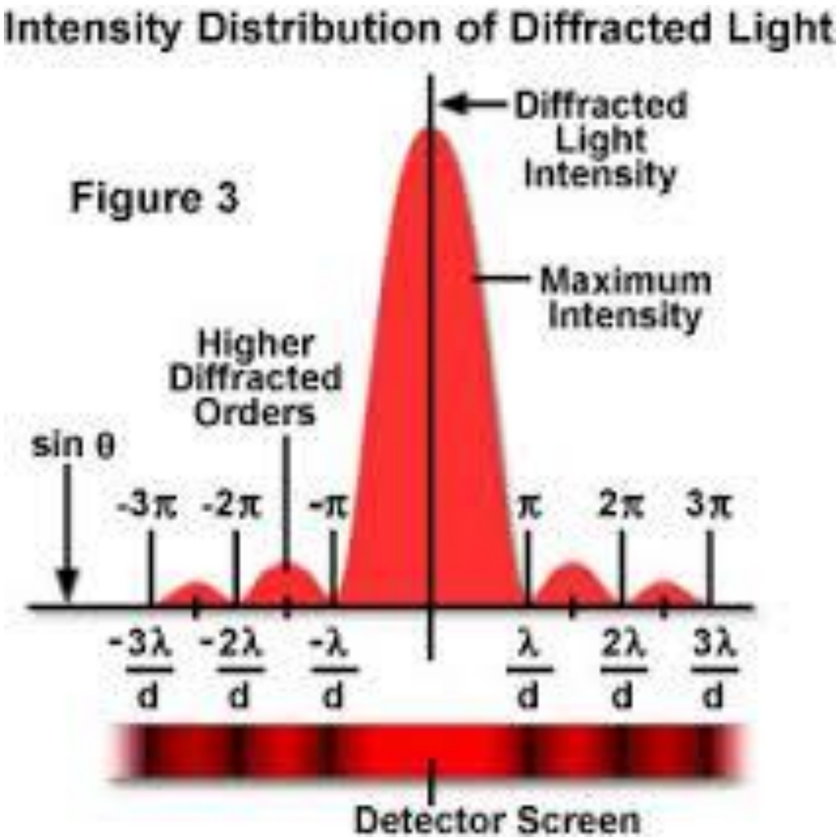
Diffraction Images – White Flash Light

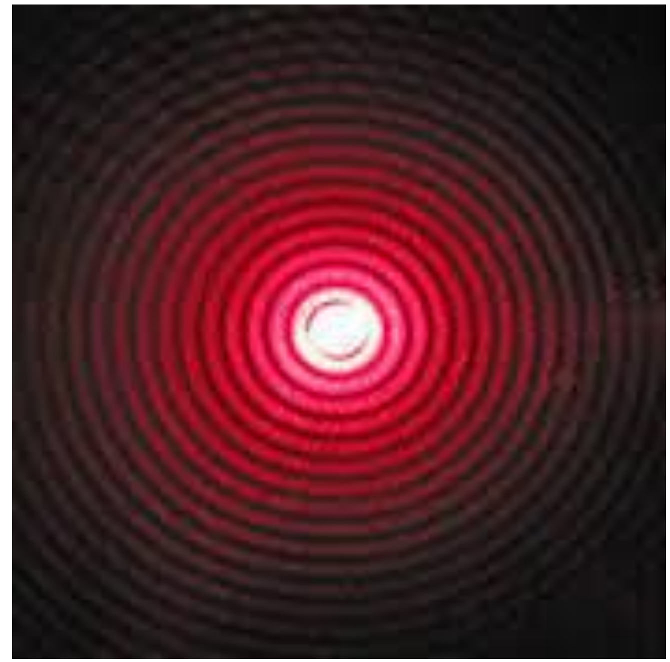


Airy Disk

Ref: Wikipedia.org

Diffraction Images – Laser Spot in the Far Field
(Laser generates a circular beam from the laser cavity)

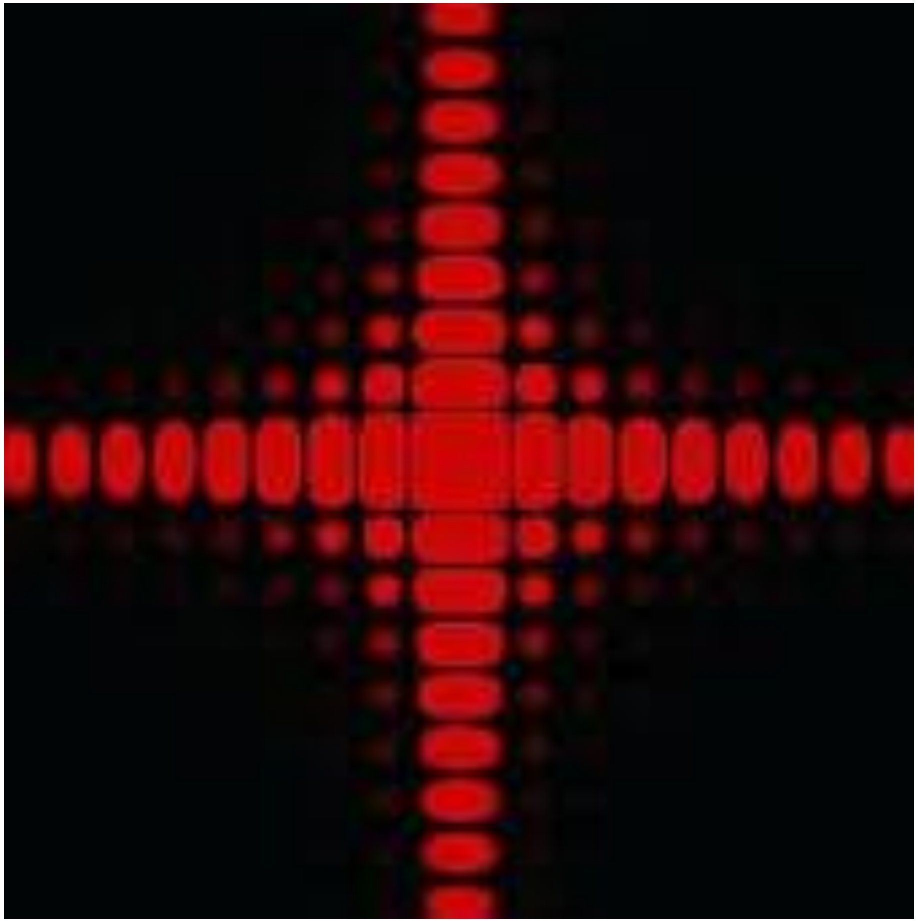




https://www.olympus-lifescience.com/en/microscope-resource/primer/lightandcolor/diffraction/

Ref: Wikipedia.org

Diffraction Images – Laser Beam Pattern Seen after Passing Through Square Aperture



https://www.wikiwand.com/en/Fraunhofer_diffraction_equation

Diffraction

Interference of waves is a mathematical idealization of the physical process of interference called diffraction.

Diffraction theory takes into account the physical limits of apertures through which the waves travel and interact to produce the interference pattern that shows special characteristics via finite edge effects.

For example, a mathematically deduced uniform interference pattern shows up as a non-uniform interference pattern representing the influence of the physical constraints of the experiment approximated by diffraction theory that indeed produces better engineering results for photonic system design.

Another way to describe diffraction is the deviation of a wave from straight line or rectilinear propagation. A simple observation of diffraction (see Figure 4.23) is the presence of a shadow made of dark and bright regions around an opaque object that obstructs light. If there was no diffraction, the shadow around the object would not exist.

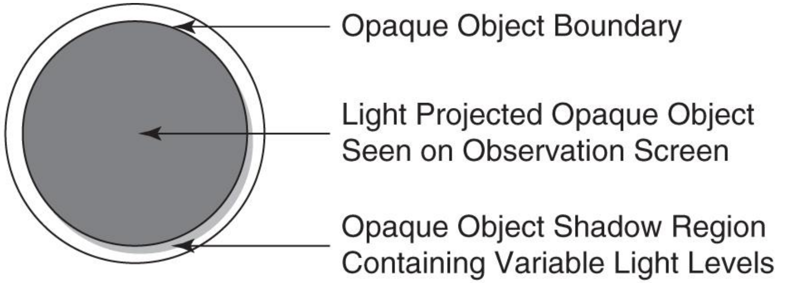


FIGURE 4.23 The light-diffraction process in action by observation of shadow region around opaque object boundary. With no diffraction, there is no shadow region.

Interference vs. Diffraction Mathematics

Interference analysis uses the sum of isolated but mutually coherent point sources on a wave front at the obstructing aperture to compute the interference spatial signal.

- For example, the Young's double-slit experiment treats the two aperture slits in the opaque obstruction as two point sources of light whose optical fields add in the far field.

- Hence, **interference studies use discrete summations of optical fields**, like *N*-fields for *N*-beam interference in a Fabry-Perot optical cavity, for example, parallel-faces optical glass block in air.

On the other hand, **diffraction studies treat the slit zone as a continuous array or distribution of point sources,** and hence **integrals a**re used to produce diffracted beam outputs.

When can one Ignore diffraction Effects

To essentially have no diffraction effects, **the aperture size in the EM wave obstructing object should be much bigger than the EM-wave wavelength** (see Figure 4.24). In this case, the plane wave through the aperture **continues its approximately rectilinear or straight line propagation**.

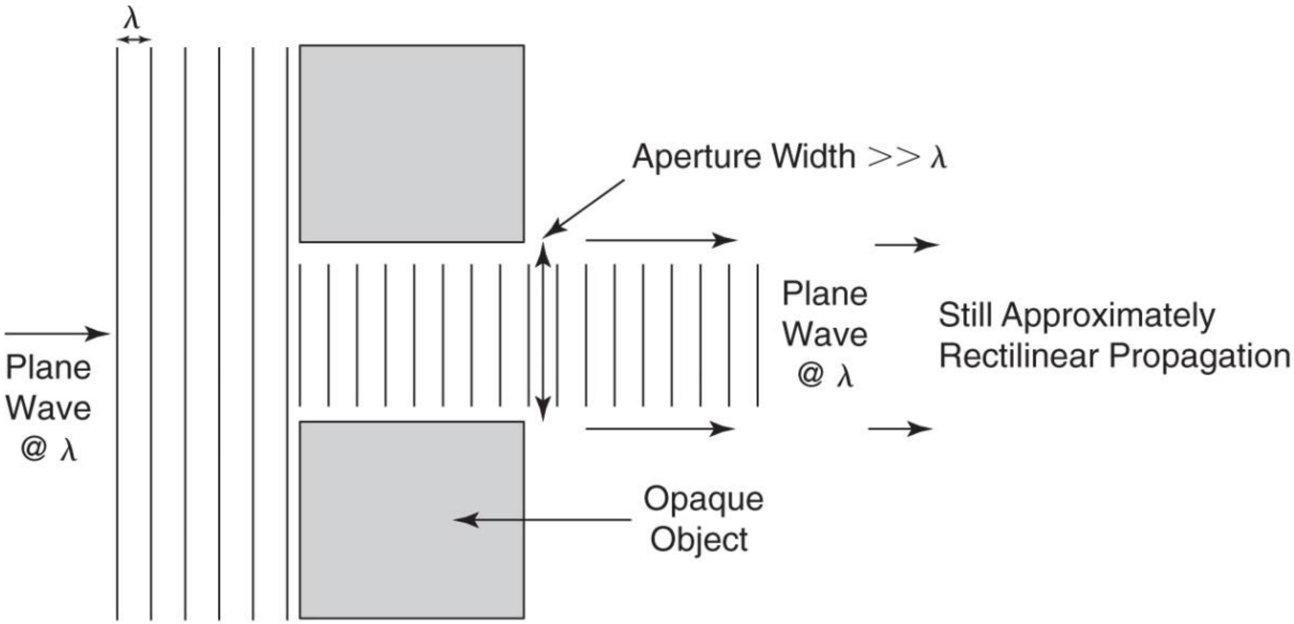


FIGURE 4.24 Conditions when light diffraction can be basically ignored to enable simpler geometrical or ray-optics based photonics system design. Hence, an input plane wave stays an output plane wave; in other words, a ray of light stays a ray of light.

Huygens-Fresnel principle of diffraction

Huygens explained **diffraction (the process of wave propagation)** by saying that every point on a given wave front can be considered as a source of secondary wavelets.

- The aperture size dictated the maximum distance between the furthest coherent point sources in the aperture wave front that interfere in the far field to produce the diffraction pattern.

Fresnel extended Huygens interpretations by adding that the actual optical field at any point beyond the wave front is a superposition of all these wavelets **taking into account both the amplitudes and phases of the wavelets.**

This presentation of diffraction is called the **Huygens-Fresnel principle of diffraction** and is an approximate theory as it does not take into account the contribution to the diffraction field of the electronic oscillators in the edges of the diffracting aperture material. These edge effects are important when the diffraction field observation point is very near the aperture.

Is the Reality Implementing Fresnel or Fraunhofer diffraction?

Apart from the wavelength compared to the aperture size, the **distance of the light source to the aperture** and the distance of the aperture to the observation point plays a key role in determining which diffraction (**Fresnel VS Fraunhofer**) approximation is suited for providing the observed diffraction pattern.

As shown in the Figure, If both the light source and observation screen are effectively far enough from the aperture so that the wave fronts arriving at the aperture and screen are plane waves, then one is in the ***far-field*** or ***Fraunhofer diffraction*** regime.

In this regime, a point source at the diffraction aperture shows up as a plane wave at the screen.

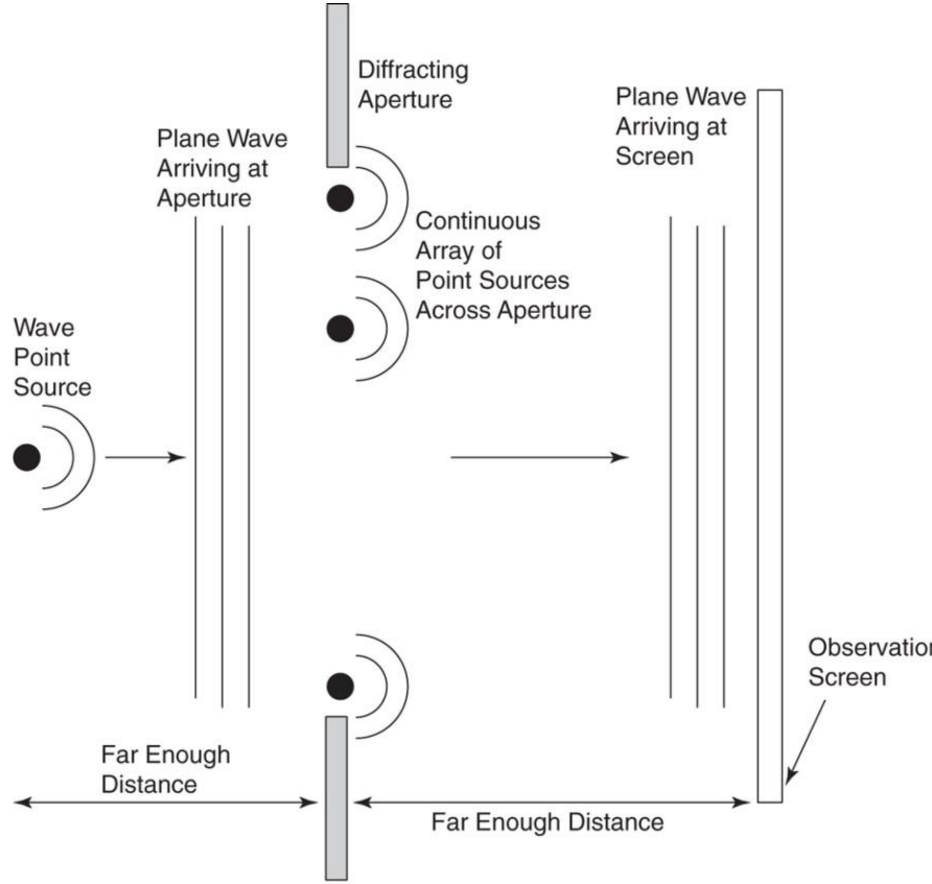


FIGURE 4.25 Conditions for far-field or Fraunhofer light diffraction where distances between source, aperture, and screen are long enough to have plane waves present at aperture and screen.

Fresnel or Near-Field Diffraction

In the *Fresnel* or *near-field* diffraction case, **the curvature of the wave front at the diffracting aperture and observation plane must be taken into account** to predict the diffraction pattern.

In this case, **both the shape and size of the diffraction pattern changes as the screen is moved while in the Fresnel regime.**

In summary, **as the screen moves away** from the aperture, the **light propagation analysis moves from the *geometric optics* regime to the Fresnel regime to the Fraunhofer regime.**

In the Fresnel regime, the distance R between source (or screen) and diffracting aperture is such that $R < (\text{aperture area})/\lambda$.

In the **Fresnel regime, one no longer uses** the **linear phase shift change** across the diffracting aperture made of continuous point sources. Now we must treat each point source on the aperture as **a spherical wave front and hence a quadratic-phase variation** of the diffracted wave across the aperture is applied to the wave propagation analysis.

Fraunhofer diffraction

As shown in Figure 4.26, Fraunhoffer diffraction assumes that the phase of the diffracting wave from the aperture varies linearly across the aperture. This is indeed the case if we draws light rays from the different source points on the aperture to the observation point and notes the linearly increasing/decreasing distances as we moves along the aperture axis.

Specifically, choose a general screen observation point D that is an x' distance above the optical axis. Pick two point sources A and B on the aperture with A treated as the reference point located at $x = 0$, while B is located a distance x away from A. Draw a plane wave front AC for the BD ray.

It is critical to assume that the distance R between the aperture and screen is large compared to aperture and screen dimensions, so $R >> x$ and $R >> x'$.

This means that the **wave propagation stays close to the optical central axis, a condition called the paraxial approximation.**

We must obey this approximation if the Fourier transform equivalent produced by Fraunhofer diffraction is to be expected on the screen.

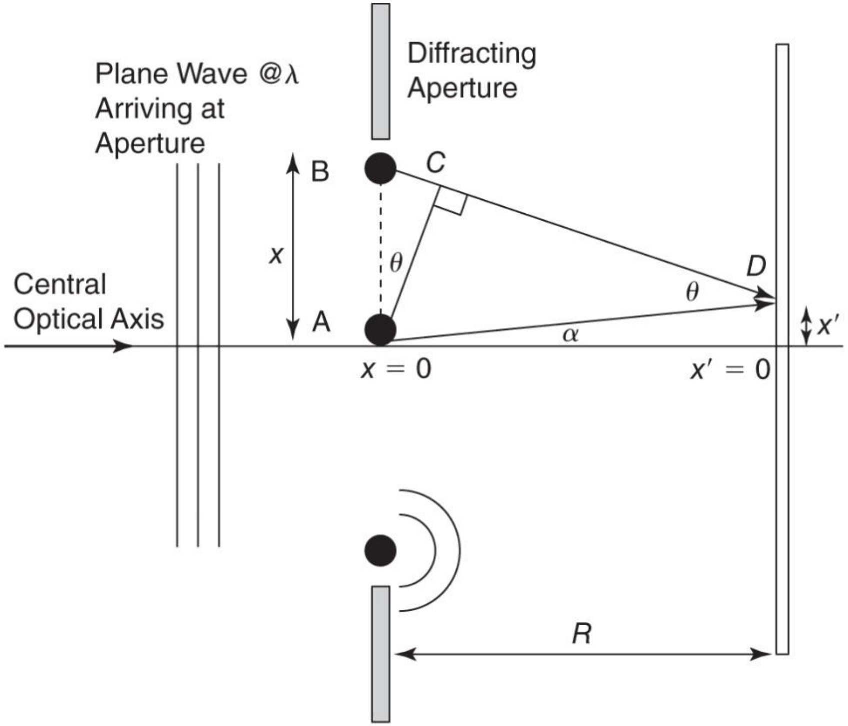


Figure 4.26 Rationale for the linear phase factor in the far-field or Fraunhofer light diffraction integral.

Fraunhofer diffraction

Using the paraxial approximation leads to several mathematical simplifications in the diffraction analysis starting with the optical path difference between the AD and BD rays is the distance BC = $x\ sin\theta$, $sin\theta \sim \theta,\ \ \alpha \sim \theta, and\ x' \sim R\theta$. **The optical-phase shift between rays AD and BD** is given by $\exp[-j(2\pi/\lambda)BC] = \exp[-j(2\pi/\lambda)\ x\ sin\theta\ ] \sim \exp[-j(2\pi/\lambda)\ x\ \theta\ ] \sim$ $\boldsymbol{exp[-j(2\pi/\lambda)\ x\ (x'/\ R\ )]}$**.** This linear phase shift factor derived in 1D Cartesian space (x to x' mapping) is embedded in the Fraunhofer diffraction integral as an exponential function. Hence, the integral giving the diffracted optical field can be written as

$$E_{\mathrm{d}}(x')\,\alpha\,\frac{1}{R}\int_{\text{aperture}} E_{\mathrm{a}}(x)\exp\left[-j\frac{2\pi}{\lambda R}x'x\right]dx$$

where $E_a(x)$ is the diffracting aperture optical field function that can be complex (i.e., has both amplitude and phase terms representing the complete aperture optical transmittance function).

One can write a spatial frequency f_x along the x' direction as $f_x = x'/(\lambda R)$,

then one can write the Fraunhofer diffracted field as

$$E_{\mathrm{d}}(f_x)\,\alpha\,\frac{1}{R}\int_{\text{aperture}} E_a(x)\exp\left[-j2\pi f_x x\right]dx$$

Note that the diffracted field expression is simply the 1D spatial Fourier transform of the aperture optical field.

1D Fraunhofer Diffraction Pattern

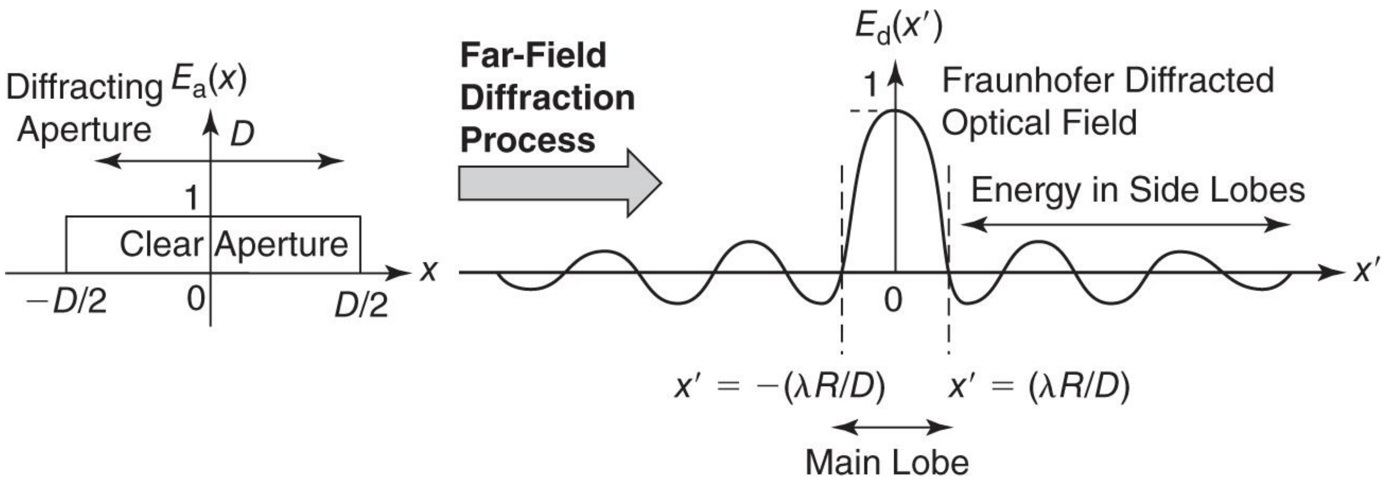


Figure 4.27 Fraunhofer optical field diffraction pattern for the given rect function diffracting aperture.

A key point to note in this Fraunhofer diffraction analysis is that the clear step edge of the optical aperture shows up in the **far field as disturbed edge with wings or side lobes where the optical energy is spatially distributed over an infinite zone**, although these contributions decrease greatly as one moves away from the central optical axis on the screen.

Fresnel or Near-Field Diffraction

We can write the 2D Fresnel diffracted field as

$$E_d(x', y') \ \alpha \ \frac{1}{R} \iint_{aperture} E_a(x, y)\, exp\left[+j \frac{\pi}{\lambda R} \{(x' - x)^2 + (y' - y)^2\}\right] \ dx \ dy$$

OR $$E_d(x', y') \, \alpha \, E_a(x, y) \ * \ h(x, y)$$

where * is the convolution operator.

Here $h(x, y) = \exp[(j\pi/R\lambda)(x^2 + y^2)]$ is the impulse response of the freespace optical system where $E_a(x, y)$ is the input signal and $E_d(x', y')$ is the output signal of the Fresnel diffracting system.

The convolution operation indicates that Fresnel diffraction is a linear shift invariant (LSI) optical system in which the impulse response of the system is a quadratic phase shift representing a 2D Huygens- Fresnel spherical wavelet.

Note that as many diffracting apertures can have complex shapes and unique amplitude and phase responses, photonic system designers using freespace propagation in a system deploy computer-based tools to compute Fresnel and Fraunhofer diffraction patterns to optimize system performance.